Effectiveness of teaching and learning in technology-supported mathematics education

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Abstract. Information and Communication Technologies (ICT tools) are the important teaching resource in modern school. Each new didactic tool is introduced into educational process in order to increase the effectiveness of this process. The paper concerns the research on the efficiency of using ICT in mathematics education. The research on effectiveness of using ICT in average secondary school in Bialystok conducted at the University of Bialystok with using research in action method is described in this paper. It was a preliminary study of what was happening at school in the classroom in the everyday didactic work with using ICT in proper didactic situations and in everyday school circumstances. Progress in using computer for change of style of work during math lesson (from drawing graphs in exercise book to concluding and structuring knowledge) was visible and quite fast. Some students noticed better results of their class tests after lessons with computer. Maybe it is a result of better understanding topics or possibility of solving more exercises instead of drawing graphs by hand. That indicates in some way the change in effectiveness of learning in new circumstances. Connection between using computers in math education and the individual development of the student and his progress in learning mathematics under the influence of using ICT in individual research work and the independent construction of mathematical knowledge will be investigated in the next stages of research.

1. Introduction

Information and Communication Technologies (ICT) mean all technologies that can process, collect and transmit information in electronic form. People should use this power primarily to build and expand their knowledge and solve problems, not just for entertainment purposes. Technology should help in creating new ideas, creating innovative projects, and the ICT potential should function every day, just like pencil and paper used to be, because today technological possibilities are simply at hand. Using ICT in mathematics education becomes more and more popular in Poland. Even if computers on the lessons are used only from time to time, not systematically, teachers observe that such lessons are more attractive for students; students are more active and motivated during these lessons.

Now the following questions are very important: How does using technology influence on effectiveness of educational process (rather – some selected aspects of this effectiveness): understanding mathematical knowledge by students and level of their skills for applying this knowledge in problems solving? What is the meaning of "students' achievements" in computer-supported teaching and learning? How should these achievements be assessed? Already in 2003

Higgins wrote that one of the goals of integrating ICT in education is to enhance teaching and learning practices thereby improving quality of education [1].

What then does "education efficiency" or "education quality" mean and how does it depend on the forms of education and didactic tools used in the process of learning and teaching, in particular on the didactic tool so popular at the moment, such as ICT?

Author of [2] asks the question: Modern technologies in mathematics lessons – benefits or waste of time? And comments this question: "A rich curriculum, a small number of teaching hours, especially when we have to prepare students for end-of-school exams, give some people the impression that using a computer in maths lessons is a waste of time because we could do several tasks at the same time. Information technology can be useful in achieving much greater goals than just automation in displaying and evaluating tasks. Its use may contribute to the student's interest in the subject, better understanding of the issues raised, and to attract the student's attention so much that he will devote much more time to the issue." [2, pp. 29-30]

Currently, during the pandemic, the use of ICT is widespread as learning takes place exclusively through e-learning. We commonly hear opinions that the effectiveness of online education is lower than the effectiveness of traditional education. In Poland, the decrease in this efficiency is considered to be so large that it is planned to reduce the scope of material in this year's state examinations at the end of primary and secondary school.

This paper is about effectiveness of using ICT in traditionally understood mathematics education: during the lessons conducted in the classroom, in the presence of teacher and students. Effectiveness of distance education, especially in such difficult and special circumstances as now, during pandemic, can be separate, very extensive topic for educational research.

2. Effectiveness of education – what does it mean?

Let's start from the aims of school education.

We can say that the aim of education is to enable students to increase the level of their competences, i.e. knowledge and skills by facilitating learning and helping in the learning process, in an atmosphere stimulating independent activity, developing student's aspirations, orienting them towards success and setting more and more difficult tasks for them, i.e. raising the bar (but always within the limits of his abilities) and introducing to critical and creative thinking.

Everyone involved in the educational process (teachers, students, parents) would like education to be as effective as possible, just like all our activities. Colloquially, we understand the "effectiveness" of an action as "a positive result, effectiveness, efficiency". On the other hand, effective teaching means the growth or maintenance of a high level of willingness (i.e. motivation) to learn, both under the guidance of a teacher and independently, and an increase in students' competences.

In pedagogical meaning, effectiveness is considered not only in connection with performance (actions) and tasks, but also in connection with wide educational aims. For many years effectiveness of educational performance in school was interpreted as improving results of teaching and learning, but now it has new meaning and is interpreted as the desired changes in students' knowledge, skills, interests and attitudes that take place under the influence of the educational process. These changes indicate the progress in student's development.

Can the use of ICT help in effective teaching of mathematics?

In order to answer this question, it is necessary to conduct research on the state of knowledge, skills and attitudes of students in ICT-supported mathematical education, but at first let's describe the main features of mathematics education supported by the use of ICT – from the point of view of practices of classroom activities.

3. Characteristics of education supported by the use of ICT

Aims of technology-supported math lesson are interdisciplinary, more complicated than the aims of traditional lesson. The aims regarding mathematical knowledge and skills should be among them as well as the aims concerning the ability of effective use information technology in mathematical problem solving. So we can expect another effectiveness of such lesson than effectiveness of traditional lesson.

Student has to be active during a technology-supported lesson – using software forces new style of work during the lesson. Teacher is not a person that knows everything and brings this completed knowledge to students; she becomes a guide in "knowledge country" that helps to construct new (for student) bits of knowledge. Student is not a "recipient of knowledge" (like during a lecture) but becomes a "constructor of knowledge" – interactive software allows making experiments and simulations, to observe, state hypotheses and verify them. So using technology seems to be the good, natural way to apply the rules of constructivism in math teaching and learning.

So student's activities during technology-supported lesson can be divided into three groups:

- activities related to computing (technological activities),
- activities related to mathematical problem solving (mathematical activities),
- activities related to planning and undertaking – with using proper software – actions that lead to solution of various mathematical problems in creative way.

In this situation, it is clear that student's achievements should be interpreted more widely than in traditional educational process. It is also clear that traditional assessment (based mostly on class tests) of traditionally interpreted student's achievements is not proper in teaching in new style. Traditional assessment leaves out of range many achievements that are formed in technology-supported math education. If student is an active "constructor of knowledge", active member of educational process, so his/her performance (especially performance with using technology) should be assessed.

Let's come back to the meaning of "effectiveness of education interpreted as the desired changes in students' knowledge, skills, interests and attitudes and consider what role can ICT play in creating effective mathematics education.

What role can ICT play in enhancing students' knowledge?

- Using interactive learning software means student's research work on solving well-defined problems leading to the construction of new knowledge.
- Searching for information in various sources is a creative way to make collections, comparisons and making conclusions – that is, to creating new knowledge.
- Using simulation software enables verifying hypotheses by students, that is very important part of creating new knowledge.

What role can ICT play in developing students' skills?

- ICT can play supportive role in skills development (e.g. facilitating task comprehension by visualizing content).
- Can enable students to solve tasks in various ways (without a computer and with a computer).
- Can play verifying role in the process of developing skills (for example solving exercises and problems).
- Can increase the attractiveness of performing exercises with the use of ICT.

How can ICT help in shaping proper students' attitudes in educational process and social life?

- Can help in shaping independence in constructing knowledge.
- Can develop creative thinking.
- Can develop critical thinking.
- Can help in shaping openness to learning with the use of ICT (not only for entertainment).

Now let's pay attention to increasing motivation students to learn that is a very important part of students' attitudes. How can using ICT help in increasing students' motivation to learn mathematics?

- Students like to work with the computer in lessons, but the teacher must remember that the computer is only a teaching resource and its use is not an end in itself.
- According to students, computer lessons are more interesting, but the teacher should remember that such lessons should provide an opportunity to satisfy cognitive curiosity, not only a curiosity about a new tool or situation.
- Using ICT makes provided content more attractive, but the teacher should remember that it should be used according to the age of students, didactic situation and the function of used tool.

4. Review of research on the effectiveness of ICT-supported education

Many different investigations on many aspects of using technology in education were and still are executed in almost all world. These investigations concern many phenomena: students' attitude to new method, their motivation for learning, change in self-confidence level, change in abilities for problem solving, sometimes investigations concern change in knowledge, understanding knowledge and skills for applying knowledge. The results of the PISA study from 2012 showed that all depends on the way new technologies are used. They are effective in shaping creativity and logical thinking, learning by creating and solving problems, also in supporting activities and in cooperation with others [3, pp. 32-34]. Research on level of knowledge and its durability in most cases were conducted as a strict experiment. Research on forming or understanding mathematical concepts are often conducted with use of very special software and/or method of case study.

Carried out research can be divided for the following groups:

(a) research on the effectiveness of the use of specific software (e.g., GeoGebra), usually conducted in the form of an experiment;

(b) research on the efficiency of using specific devices (laptops, iPads), specially created learning environments or ICT as a whole (i.e. different devices and/or different software);

(c) research on effectiveness of distance education (blended learning, e-learning).

References to selected research reports are provided below.

Ad (a)

Generally conclusions from the research on on the effectiveness of the use of specific software are positive.

The results of study on the effectiveness of Geometer's Sketchpad, a Dynamic Geometrical Software (GSP), in facilitating the teaching and learning of Mathematics, among technical schools students, described in [4] shown that the empirical tests have indicated the effectiveness of GSP in the teaching of mathematics through the substantial performance differences in favour of the experimental (taught with using of GSP) group. The results also shown that GSP has helped the students to better retained the knowledge taught.

The results of study on effectiveness of using GeoGebra software on mathematics learning among 62 students in Malaysia, described in [5], shown that students have positive perception towards learning and have better learning achievements using GeoGebra.

In [6] long three-year investigation on effects of using GeoGebra in fifth grade of rural school in Hungary is described. GeoGebra was used there in teaching of some topics from geometry and also for basic operations with integers and fractions. What is interesting: results of experiment showed that in case of using GeoGebra for operations on integer and fractions the results of experimental group have been shifted towards the positive direction, but not in so large scale as in the case of geometry. It means that in case of specific software its using should be very carefully planned according didactic situation.

Investigation from India is described in [7]. The aim of this study was to examine the impact of using the free educational software GeoGebra on 9th grade students' mathematics achievement in learning geometry. A total of 50 students were selected from a government school located in the eastern part of India. The experimental group (25) was taught theorem on circles using GeoGebra while the control group (25) was taught utilizing traditional teaching methods. At the end of the treatment, students‘ mathematics achievements were measured using a post - test. The result indicated that GeoGebra is an effective tool for teaching and learning geometry in middle school.

Ad (b)

Here results are not so obviously positive.

In [8] author writes: "Results showed that students who used laptops in class spent considerable time multitasking and that the laptop use posed a significant distraction to both users and fellow students. Most importantly, the level of laptop use was negatively related to several measures of student learning, including self-reported understanding of course material and overall course performance."

In [9] authors describe the experiment conducted with virtual space world (ANIPPO environment created for solving problems from 3-dimensional geometry) in teaching geometry in fifth grade. The results are the following: "ANIPPO world seems to be an obstacle to solve the problems. Technical problems have disturbed the experimentation (domination of technic on didactic). Experimental group had not sufficient time to appropriate ANIPPO world and it is a too complex appropriation. Complexity of inter-registers treatment in ANIPPO world: long text written in natural langages on the screen, complex keyboard manipulations to move in the ANIPPO world, confused oral exchanges between pupils. ANIPPO world with avatar plays more a distractor role than a motivating role." Also authors highlights difficulties of problems interpretation : space geometry is not enough taught in primary school, problems were too difficult, Protocol of pre-experimentation was disturbed by technical problems. What does this study say to us, teachers? Our conclusion is that not always modern, specially created learning environments are the best solution for students.

New technological gadgets, like iPads, smartphones etc. appeared in educational space and of course some investigations on their effectiveness as didactic tools were conducted. In [10] very interesting research on innovative approach to exploring student interaction with iPad apps is described. It focuses specifically on design and content features of apps selected by an experienced teacher to enhance literacy, numeracy and problem-solving capabilities of her 5 year old students. Findings reveal a complex matrix of influencing factors. These include the effect of embedded pedagogical scaffolds (eg., modelling, reflection time), corrective and formative feedback, text-to-speech functionality, imposed interaction parameters, impediments (eg., web links, advertisements, buying content) and the entertainment/education balance. Arguments are made for researchers, teachers and developers to work together and adopt methodologies such as that introduced in this article, to gather data to radically improve the design of apps used by young students for learning.

Very interesting experimental course called "iTrust: Centralized, Distributed, and Mobile Search" is described in [11]. In this course, 22 high-school students needed to understand the differences between centralized and distributed search engine, identify their trade-offs, explore

the iTrust gadgets (desktop, cell phones, and tablets), in order to understand simple statistical equations. As the conclusion the author writes: "My pedagogy allows my students to become more active in the class, to obtain more learning knowledge, to increase their learning motivations, to pay greater attention in my class, and ultimately to simulate their interests in learning inside and outside of the class. As a result, it encourages students to engage more actively in the class and lead to greater learning in the classroom. In addition, once the students become engaged, instructors will become more motivated toward teaching."

Case study of developed a computer-aided instruction utilizing the Scratch program, which is a programming tool, and a mathematics curriculum with Scratch program, and applied the developed curriculum to teach mathematics as a recipe of a practical instruction for the 21st century skills and positive attitude toward mathematics is described in [12]. The result of this case study shown that this intervention has a great possibility as an alternative way to teach mathematics in ways that stimulate learners' various abilities, such as creativity, problem solving, logical thinking and the like, as well as that build a positive attitude toward mathematics.

A Meta-analysis of the Effects of Computer Technology on School Students' Mathematics Learning is described in [13]. This study examines the impact of computer technology (CT) on mathematics education in K-12 classrooms through a systematic review of existing literature. A meta-analysis of 85 independent effect sizes extracted from 46 primary studies involving a total of 36,793 learners indicated statistically significant positive effects of CT on mathematics achievement. In addition, several characteristics of primary studies were identified as having effects. For example, CT showed advantage in promoting mathematics achievement of elementary over secondary school students. As well, CT showed larger effects on the mathematics achievement of special need students than that of general education students, the positive effect of CT was greater when combined with a constructivist approach to teaching than with a traditional approach to teaching, and studies that used non-standardized tests as measures of mathematics achievement reported larger effects of CT than studies that used standardized tests.

Ad (c)

Research on effectiveness of distance education outpaced the really extensive use of e-learning in education – pandemic in 2020 forced almost all countries to change the form of education from stationary to distant. Probably very serious research on effectiveness of such form of education will start now, so here only few examples of investigations from the previous years.

Blended Learning: Does it help students in understanding mathematical concepts? - the author of [14] asks. This experimental research aimed to investigate the effectiveness of learning mathematics delivered through blended learning. In particular, this study compared the effectiveness of blended learning using Moodle and traditional one in relation to students' conceptual understanding. The data was gathered through written test and interview. 127 students of grade eight secondary school were involved. The data shows that students who learn in blended learning approach have better conceptual understanding rather than their counterparts. The students claim that they can access learning material and revisit some difficult material in their convenience time. Nevertheless, there are some obstacles faced by students related to maintaining their motivation to learn independently and keeping distraction off to access other website when they learn online.

What drives a successful e-Learning? – the author of [15] asks. Study reported in the article developed an integrated model with six dimensions: learners, instructors, courses, technology, design, and environment. A survey was conducted to investigate the critical factors affecting learners' satisfaction in e-Learning. The results revealed that learner computer anxiety, instructor attitude toward e-Learning, e-Learning course flexibility, e-Learning course quality, perceived usefulness, perceived ease of use, and diversity in assessments are the critical factors affecting learners' perceived satisfaction. Study did not concern only mathematics education, but its results (although not very new) are very important now, in the days of forced universal e-learning.

At the end of this research review, it is worth quoting the conclusions and recommendations of systematic review and literature survey of research conducted from 1985 until 2015 described in [16]: "Based on the summary of almost 30 years of research, this study provides important conclusions related to the effectiveness and moderators of technology integration in mathematics classrooms. In conclusion, the results of this systematic review indicate that technology integration supports mathematics achievement across prior meta-analytic research. (...) Based on these results, the researchers recommend that teachers and researchers continue to implement technology in the mathematics classroom, but emphasize optimizing the effects grade level, role of technology, and duration. (...) In addition, more research is necessary to capture the unique influences of teachers on the effects of technology integration in the mathematics classroom."

5. Example of research on the effectiveness of ICT-supported education conducted at the University of Bialystok

Presented reports from research show that there is small number of research works on effectiveness of teaching and learning with using computers conducted in regular, systematic work with a whole class, not in experimental conditions, but in everyday school circumstances.

In this situation at the University of Bialystok was made decision to start with such research because:

- Computers (with proper educational software) should become – and in many cases become – a didactic tool not only for individual work, but also for a common work in a classroom.
- All students – working in normal school circumstances – should profit from using computers in their learning, so research on results of using this tool in the whole math education, with all obstacles that school reality brings, should be done.
- During technology-supported math lesson students have opportunity to develop some special abilities, so the ways of assessment should take into consideration this change in students' achievements.

5.1. Obstacles

There were a few difficulties in planning and conducting research of above-mentioned kind:

- Small number of math lessons per week; in some types of schools there are two hours of mathematics per week, so teachers very often have to make hard decision: to devote lesson for practice in doing traditional exercises or to devote lesson for using computer (that is in mind of many people considered rather as a play).
- Not easy access to computer laboratory; there are many lessons of ICT in the schools, so labs are occupied, but in many schools situation becomes better because of the second multimedia lab that schools are given just for using technology in teaching all school subjects.
- Teachers' abilities to use technology in their work with students; many teachers (not all, but many) don't trust themselves, their abilities.
- There is a paradox in Polish educational reality: In the national curriculum it is written: "The most important skills developed in general primary education are: (...) creative problem solving in various fields with the conscious use of methods and tools derived from computer science" [17]; "The most important skills acquired by a student during general education in general secondary school and technical secondary school include: (...) creative problem solving in various fields with the conscious use of methods and tools derived from computer science, (...) the ability to efficiently use modern information and communication technologies." [18], but using all technological tools is forbidden during the exams (except of the simplest calculator during the exam after secondary school). Results of the exams are "the gate" to school of higher level, so teachers are expected (at first and most important) to

prepare students for exams. The conclusion for many teachers is simple: if technology is not used during the exams, so why am I to waste time, waste lessons for going to computer lab? But I must say that many teachers try to save time on the lessons for practice for exams and, in the same time, try to use additional hours for work in the laboratory. Teachers that teach ICT and mathematics are in better situation: they can teach bits of mathematics with computers on ICT lessons.

Very important remark: The use of multimedia textbooks by displaying them on the multimedia board and using the multimedia board for writing instead of the usual blackboard is not considered to be using ICT in math lessons. We took into consideration only the use of computer as the tool for investigative work of students.

5.2. Aims of the study

The main aims of the study were:

- introducing the elements of assessment of students' work on the computers to technology-supported math education,
- making research on the influence that using technology and assessing students' work on computers have on some aspects of effectiveness of teaching and learning process,
- recognition students' and teachers' opinions about the effectiveness of using systematic use of computers as the tools of constructing knowledge by students and about new way of assessing in math education.

From among problems that are included into the idea of "effectiveness of educational process" the following ones have been chosen to investigation:

- taking up by students active and creative position in technology-supported learning,
- the range of students' own work on problem solving with the use of technology (work at home and everywhere outside classroom is included),
- taking up by students position of responsibility for the results of their learning and development: their own planning and consistent accomplishment of their educational objectives,
- formation (structuring) knowledge by students based on the results of their work with computers.

5.3. Research problems

The following main research problem has been formulated: **What kind of influence has introducing the alternative assessment into technology-supported math education on effectiveness of math teaching and learning?**

The main problem has been divided into the following detailed problems:

- What is the influence of introducing assessing abilities to use technology in learning on students' creative activity?
- Does introducing assessing abilities to use technology in learning cause changes in students' attitude toward their learning?
- How does the constructing knowledge by students in technology-supported math education progress?

5.4. Research methods, techniques and tools

Action research method and an exploratory probing have been chosen as the methods of investigation. Questionnaire and document analysis has been chosen as research techniques. As investigation tools questionnaires for teachers or students and a number of special form of homework after the lessons in which computers were used. Also all students' documents – in paper or electronic form – created during the lessons or outside classroom are collected in portfolios and used as documents for analysis.

No hypotheses were formulated. This is a result of choice of action research as a main research method [19]. Researchers choose this method when they want to investigate particular phenomenon in the reality, in the full context and get to know them just as they are, not find that they fit to our notion about them. This method allows taking up for investigation problems that are almost unknown, such problems that researcher rather has some intuition than knows something about them, so it is difficult to formulate hypotheses. In case of considered research, the effectiveness of math teaching and learning in context of using technology and assessing the results of students' work with computers. Of course, we would like to observe the positive influence of these elements on effectiveness of educational process in many respects, but we cannot assume this. At the beginning of the research we don't know what school reality will bring. So researcher rather wants to observe and describe changes in students' development than try to fit the results of investigation to my assumptions and expectations.

5.5. Procedure of investigation

Investigation started in two classes (totally 56 students) in second grade of secondary economical school in Bialystok. There were two math lessons per week in each class, so it was clear from the beginning that it would be impossible to have many lessons with use of computers during a school year. In these classes mathematics was taught on basic level. Students from selected classes did not work with computers on math lessons before the beginning of the project.

Before the beginning of school year we (author of this paper and math teachers from selected classes) have looked at the plan of lessons for the whole year and chose the topics that could be supported by using computers and proper software. We realized that most of topics (accordingly to curriculum) regard functions, so we chose program *Graphic Calculus* (authors: Piet van Blokland and Carel van de Giessen from VuSoft, Netherlands [20]) as software for selected lessons. During the first math lesson in the computer laboratory students learned the program, then they could make copies for their home computers.

During all school year it was possible to have THREE technology-supported math lessons in computer lab with each class. Math teachers of these classes cooperated with each other in preparation materials for the lessons. All students from two classes had to make the same homework after each lesson.

5.5.1. First lesson (In case of this first lesson the entire plan is given here, in the next cases I will give the short descriptions)

Topic: Graphs of polynomial functions

(Note: This lesson comes after the lesson on topic: Polynomial equations *and precedes the lesson on topic:* Polynomials inequalities. *Students can already find zeros of polynomials – solve the equation* $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0 = 0$. *In each lesson about polynomials we use the formula of polynomial like in previous sentence. We teach solving polynomial inequalities by finding zeros of polynomial, drafting a simplistic graph and reading the signs of polynomial values from this graph.)*

Age of students: 17

Duration of the lesson: 45 minutes.

Aims:

- general:
  - to improve the ability of using Graphic Calculus,
  - to enlarge knowledge about polynomials,
  - to form a positive attitude to solving problems and inquiring,
  - to facilitate accurate, logical and critical thinking,
- detailed – after the lesson students can:
  - describe how the graph of polynomial function depends on its zeros,
  - describe how the degree of polynomial and the sign of coefficient a_n influence the polynomial values in infinity ($-\infty$ and $+\infty$),
  - draft a graph of polynomial knowing its zeros and sign of coefficient a_n.

Methods: discussion, brain storming session, work with computers, problem solving method.
Forms of work: plenary session and work in pairs (with computers).
Procedure of the lesson:

- Introductory part:
  Students revise:
  - the idea of "zero of polynomial function",
  - the connection of the zero of any function with its graph,
  - the rules of using *Graphic Calculus*, especially needed on the lesson option *Draw Graphs*.
- Main part:
  Students work with computers in pairs. Before working with computers students are given their task: they have to observe what the influence of degree of polynomial, multiple of zeros and sign of coefficient a_n on different attributes of its graph is.
  They use GC: draw and watch graphs of polynomials:
  $y = (x-1)(x-2)(x-3)$ – *three single zeros, odd degree of polynomial,* $a_3 > 0$
  $y = -3(x-1)(x-2)(x-3)$ – *three single zeros, odd degree of polynomial,* $a_3 < 0$
  $y = (x-1)^2(x+3)$ – *one single zero, one double zero, odd degree of polynomial,* $a+3 > 0$
  $y = -2(x-1)^2(x+3)$ – *one single zero, one double zero, odd degree of polynomial,* $a_3 < 0$
  $y = -2(x+3)^3(x-2)^2$ – *one triple zero, one double zero, odd degree of polynomial* $a_5 < 0$
  (Note: The teacher or students can build other examples of polynomials. The form of these polynomials saves time needed to find zeros and help to observe the connection between zeros and graph.)
  We hope that after watching the graphs students will conclude:
  - The polynomial's graph is a curve, which intersects the X-axis in points $(x_i, 0)$ if x_i is a zero of odd multiple, and reflects on the X-axis in points $(x_j, 0)$ if x_j is a zero of even multiple.
  - If $a_n > 0$ then polynomial values in infinity (when $x \to \infty$) are positive.
  - If $a_n < 0$ then polynomial values in infinity (when $x \to \infty$) are negative.

  These conclusions will help students to draft the graphs of polynomials when solving polynomial inequalities.
  Students should write down their conclusions in their exercise books and try to draft the graphs of the following polynomials by themselves:
  $y = 2(x-1)(x-2)^2(x-3)$
  $y = -3(x-1)^2(x+2)^3(x-3)$.
- Final part:
  1. Short recapitulation of the lesson.

2. Getting the feedback from the students by asking questions: Is everything clear? What was the main difficulty?
3. Setting the homework.

Many years ago author of this paper created the special form of homework after lessons with computer. This homework consists of two parts: first part plays role of feedback, second part contain exercises for practice related to topic of the lesson. Students like this form of homework, because they know that their opinion about the lesson is important for teacher and they feel responsibility for some part of math education.

After first lesson with computers students received the following form:

Name: _____ Date: _____

Homework after the lesson on topic: Graphs of polynomial functions
Answer the following questions:

1. What was the lesson about?
2. What were you doing during the lesson? Describe your activities.
3. Was using computer during this lesson useful? Justify your answer.
4. What was good about the lesson?
5. What was bad about the lesson?

Draft the graphs of following functions:

- $f(x) = -2x(x-3)^3(x-5)^2$,
- $f(x) = 4(x+5)(x+4)(x+3)(x+2)(x+1)$

Results of first lesson
All students wrote that using computer had been useful because graphs of functions had been made correctly, precisely and fast.
The table 1 shows students' answers questions No 4. and 5.
Some conclusions from homework:

- Many students wrote: **"It was possible to understand the topic faster and easier"**. In the same time about 80% students did not solve exercises (did not draft two graphs) correctly. Conclusion is: students' knowledge after the lesson was very weak; they learnt almost nothing about the connection between zeros and graph of polynomial. The short-term effectiveness of this lesson was very low.
- Many students wrote: **"Computer did all work for us"**, and assessed this as the wrong part of the lesson. Conclusion is: Students consider math activities only as manual actions, like drawing graphs.
- Also they wrote: **"We had to conclude from graphs by ourselves"** and, mostly, assessed this as the wrong part of the lesson. Conclusion is: Students are not used to put hypotheses and make conclusions. They are not used to construct knowledge. They are a little bit lost when computer does all "manual work" and leaves them thinking.
- Some students copied their homework from each other.

It became clear that students were the beginners in using computers for learning and structuring knowledge, so we (I and math teachers) decided to take the results of first lesson as a point of reference and observe a progress just from this level.

Table 1. Students' opinions about good and bad sides of the lesson.

Good sides of the lesson	Bad sides of the lesson
A graph has been made immediately after typing a formula.	**I could not learn too much because computer did all work for me.**
Cooperation.	**It was necessary to commit myself very much.**
Easy examples.	Not enough number of computers in the lab.
New experiences.	**It was difficult to determine properties of function.**
Learning computing.	**It was difficult to conclude from graphs.**
It was possible to understand the topic faster and easier.	**We had to conclude from graphs by ourselves.**
Mathematics with computers is more interesting.	**Lack of skills in making conclusions.**
Lesson was quite different from typical math lessons.	**Computer did all work for us (we had only to type formulas).**
Fast pace and precision of work.	
It was like a play.	
We were mobilized to conclude from graphs.	
It was possible to compare a few graphs in the same coordinates system.	
Computer did work for us.	

5.5.2. Second lesson Topic: Graphs and properties of functions $y = \sin x$ and $y = \cos x$

Procedure of the lesson:

Students were working with program *Graphic Calculus*, options *Trigonometric Functions* and *Draw Graphs*. They created graphs of functions $y = \sin x$ and $y = \cos x$, discussed the main properties of these functions. Then, in groups, students had to solve the following exercises:

Exercise 1.

Create (using software) graph of function $y = \sin x$ and read from the graph:

a) range of function values,
b) zeros of function,
c) intervals in which values of the function are positive,
d) arguments for which value of the function equals 1,
e) arguments for which value of the function equals –1,
f) intervals in which function is decreasing.

Exercise 2.

Create (using software) graph of function $y = \cos x$ and read from the graph:

a) the lowest and the highest value of the function,
b) range of function values,
c) zeros of function,

d) intervals in which function is increasing and intervals in which function is decreasing,
e) line of symmetry of the graph,
f) arguments for which $\cos x = 1, \cos x = 0, \cos x = -1, \cos x = 0.5, \cos x < 0, \cos x > 0$.

Write down results of your work in your exercise book.

Homework after this lesson:

Name: _____ Date: _____

Homework after the lesson on topic: Graphs and properties of functions $y = \sin x$ and $y = \cos x$

Answer the following questions:

- What was the lesson about?
- Was using computer during this lesson useful? Justify your answer.
- What was good about the lesson?
- What was bad about the lesson?

On this sheet draft (by hand) graphs of functions:

1. $y = \cos x$,
2. $y = \sin x$.

and write down answers the questions:

. . . . . .

(Questions were similar to questions from the lesson)

Some conclusions from homework:

- Only one student wrote that using computer on this lesson was not useful, many other students noticed two good things: possibility of observation the process of rising graphs and possibility of work with radians and degrees. Conclusion is: students want to understand the processes, not only the results.
- Two students wrote: "Computer did all work for me, it was bad in the lesson". Conclusion: Still some students do not count thinking, reading from the graph, concluding, discussing etc. as the mathematical activities.
- All students made exercises, although some of them made mistakes. Conclusion: the effectiveness of the lesson was better than in case of first lesson with computer.

5.5.3. Third lesson Topic: Transformations of graphs of trigonometric functions

Procedure of the lesson:

Students were working in pairs with program *Graphic Calculus*, option *Draw Graphs*. They had to create the proper graphs and fill in the table with the results.

Example of the task:

Basic function: $y = \sin x$. Formula of function after transformation: $y = (x - \pi/3)$ – task for students: to create graphs of both functions, to describe the transformation and to give the domain and range of values of function after transformation.

After work with computers students formulated the conclusion about the connection between the kind of transformation (the direction and distance of translation) of graph and the formula of obtained function.

Homework after this lesson contained the questions about the lesson and two exercises that had to be solved on the sheet of homework:

Exercise 1

Draft graphs of functions: $y = \cos(x - \pi/4) + 1$, $y = \tan(x + \pi/3)\check{}2$ (and some others)

Exercise 2

Write down the formula of function that you obtain after translation the graph of function y=cos x four units down

(and a few similar examples)

Conclusions from homework:

- Students are used to work with the program. They do not complain that concluding is hard, but are happy that computer allows comparing graphs and helps – by visualization – to remember the mathematical rules.
- Students solve exercises (sometimes with mistakes) by themselves.

6. Assessment

During all lessons students had opportunity to be given good mark for active participation in the lesson. Homework was not assessed in marks.

After third lesson eager students had opportunity to take additional tasks (related to the topics) for their own work – for good mark. Only 14 students from one class and nobody from the other class decided to do this additional work.

All homework, sheets with tables completed during the lessons and additional works became a parts of students' portfolios.

7. Questionnaires

At the end of school year students completed the questionnaires. They were asked for answering some questions related using computers in math education. Only 38 students from two classes have completed the questionnaires (it is only 68% of all students involved in investigation), so in the results presented bellow $n = 38$.

The answers to the questions from the survey are presented below.

- Question: What results of using computers in math education do you observe in yourself? – *Students could choose and mark items from the list.*
- Question: Is it necessary to keep the results of students work with computers in any form (notes, printouts, files on hard disk)?
  Answers:
  Yes – 28 students, 73.7%
  No – 7 students, 18.4%
  No answer – 3 students, 7.9%
- Question: Should students' work on computers during math lessons be assessed?
  Answers:
  Yes – 21 students, 55.3%
  No – 14 students, 36.8%
  No answer – 3 students, 7.9%

Some conclusions from the questionnaires:

Most of participating in the survey students indicated that it is easier for them to understand topic with using software in learning, but only few of them noticed connection between using ICT and better results of tests.

Table 2. Results of using computers in math education observed by students in themselves.

Item from the list	Number of students	Percent of choices
Bigger motivation for learning	13	34.2
Bigger activity	12	31.6
Easier understanding the topic	30	78.9
Bigger abilities to problem solving	24	63.2
Easier concluding, structuring knowledge	19	50.0
Better results of class tests	9	23.7
Other		

More than 50% of participating in the survey students believe that their work with computer should be assessed, but only few of them used occasion to receive good mark for additional task with computer.

What does it mean? It is big question for teachers. Also it is some hint what direction of further work should be.

8. Conclusions and directions of further work

The first stage of introducing new didactic tool to math education in two classes from average secondary school in Bialystok has been described in this paper. Even this not-too-big experience was very informative for us (author of the paper and math teachers), especially as for progress in using computers for structuring the knowledge and students' attitude towards giving marks for the results of work with computers.

Let's remain research problems and conclude what answers these problems the results of the research gave:

- What is the influence of introducing assessing abilities to use technology in learning on students' creative activity? – Answer: 63% of responding students said in the survey that using ICT gave them bigger abilities to problem solving, 50% of responding students said that concluding and structuring knowledge was easier with computer. It is not too big number, but it is necessary to take into consideration that the period of shaping these students' abilities was not too long (only three lessons).
- Does introducing assessing abilities to use technology in learning cause changes in students' attitude toward their learning? – Answer: It is difficult to answer definitely, especially that most of students did not use occasion to do additional task even for good mark.
- How does the constructing knowledge by students in technology-supported math education progress? – Answer: here situation is rather clear: students were better in constructing knowledge at each subsequent lesson – it was visible from their homework.

Only partial answers the research problems were received, but we do not worry because the research was introductory research on this what is going on in the classroom in the everyday didactic work with using ICT in proper didactic situations and in everyday school circumstances. The most important for us was the following:

- Students accepted computer and software not only as a tool that can increase attractiveness of lesson, but also as a tool for learning.
- Progress in using computer for change of style of work during math lesson (from drawing graphs in exercise book to concluding and structuring knowledge) was visible and quite fast.

- Many students understand that the results of their work with computer should be saved somewhere in some form; they accepted the portfolio idea.
- About 50% of responding students agree that the results of their work with computer should be assessed in school mark and this mark should influence the final grade. But, in the same time, the opportunity to receive good mark used only several students.
- Some students noticed better results of their class tests after lessons with computer. Maybe it is result of better understanding topics or possibility of solving more exercises instead of drawing graphs by hand. That indicates in some way the change in effectiveness of learning in new circumstances. Connection between using computers in math education and results of class tests and exams will be investigated in the next stages of research. It is hard task; special tools should be constructed.
- Math teachers from selected classes' notices that shy students became more self-confident after introducing computers into learning process. Possibility of work at home with computer on homework assignments and possibility of good, creative preparing homework was very encouraging for students, especially if students' skill for computing were better that for math.
- One more result of introducing technology into math education appeared: cooperation between students and ICT teacher and cooperation between math teachers and other subjects' teachers (statistics, economics, ICT). We did not expect this result and now we are very pleased.

Students want more math lessons with computers. Research that has been started will be continued in the future, after coming back to normal work in schools after pandemic – we hope that it will be possible next school year; three lessons are not enough to answer the research problems. Maybe investigation will be expanded because number of math teachers from other schools would like to join this research. We will focus on the individual development of the student and his progress in learning mathematics under the influence of using ICT in individual research work and the independent construction of mathematical knowledge.

References

[1] Higgins S 2003 *Does ICT Improve Learning and Teaching in Schools?* (Nottingham: British Educational Research Association)
[2] Kwiecień D 2020 *Efektywne metody nauczania matematyki dla uczniów gimnazjów i szkół ponadgimnazjalnych z wykorzystaniem TIK* (Warszawa: Ośrodek Rozwoju Edukacji)
[3] Janczak D and Grześlak M 2016 *Rozwijanie kreatywności i postaw proinnowacyjnych uczniów z wykorzystaniem nowych technologii* (Warszawa: Ośrodek Rozwoju Edukacji)
[4] Ramli R, Mustapha R and Ramli R 2014 A study on the effectiveness of a technology supported approach in the teaching of mathematics — Using Geometers' Sketchpad (GSP) *2014 2nd International Conference on Technology, Informatics, Management, Engineering Environment* pp 105–110
[5] Arbain N and Shukor N A 2015 *Procedia - Social and Behavioral Sciences* **172** 208–214
[6] Budai L 2011 *Annales Mathematicae et Informaticae* **38** 129–136
[7] Bhagat K K and Chang C Y 2015 *Eurasia Journal of Mathematics, Science and Technology Education* **11** 77–86
[8] Fried C B 2008 *Computers & Education* **50** 906–914
[9] Pacurar E, Magot C A, Cabassut R and Solon Y 2020 *Teaching Mathematics and Computer Science* **18** 229–240
[10] Falloon G 2013 *Computers & Education* **68** 505–521
[11] Chuang Y T 2014 *Creative Education* **5** 1969–1978
[12] Choi B, Jung J and Baek Y 2013 In what way can technology enhance student learning? : A preliminary study of technology supported learning in mathematics *Proceedings of Society for Information Technology & Teacher Education International Conference 2013* ed McBride R and Searson M (New Orleans, Louisiana, United States: Association for the Advancement of Computing in Education (AACE)) pp 3–9 URL `https://www.learntechlib.org/p/48061`

[13] Li Q and Ma X 2010 *Educational Psychology Review* **22** 215–243
[14] Setyaningrum W 2018 *Jurnal Riset Pendidikan Matematika* **5** 244–253
[15] Sun P C, Tsai R J, Finger G, Chen Y Y and Yeh D 2008 *Computers & Education* **50** 1183–1202
[16] Young J, Gorumek F and Hamilton C 2018 *Journal of Computers in Education* **5** 133–148
[17] 2017 Rozporządzenie Ministra Edukacji Narodowej z dnia 14 lutego 2017 r.w sprawie podstawy programowej wychowania przedszkolnego oraz podstawy programowej kształcenia ogólnego dla szkoły podstawowej, w tym dla uczniów z niepełnosprawnością intelektualną w stopniu umiarkowanym lub znacznym, kształcenia ogólnego dla branżowej szkoły I stopnia, kształcenia ogólnego dla szkoły specjalnej przysposabiającej do pracy oraz kształcenia ogólnego dla szkoły policealnej URL `https://podstawaprogramowa.pl/files/D2017000035601.pdf`
[18] 2018 Podstawa programowa kształcenia ogólnegodla czteroletniegoliceum ogólnokształcącegoi pięcioletniego technikum URL `https://archiwum.men.gov.pl/wp-content/uploads/2018/01/zalacznik-nr-1.pdf`
[19] Zaczyński W P 2006 *Praca badawcza nauczyciela* 6th ed (Warszawa: Wydawnictwa Szkolne i Pedagogiczne)
[20] van Blokland P and van de Giessen C 2019 VuSoft - Home URL `https://www.vusoft.eu/`